MODEL ANSWERS

AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
MATHEMATICS
Paper 3

Exam Date Morning Time allowed: 2 hours

Materials
For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

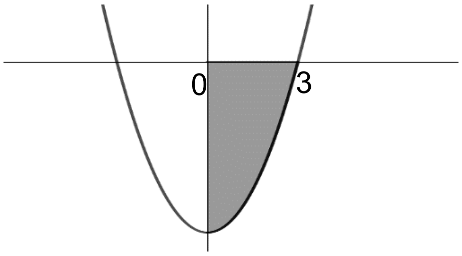
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

1 The graph of $y = x^2 - 9$ is shown below.



$\int_0^3 x^2 - 9 \, dx = \left[\frac{1}{3}x^3 - 9x\right]_0^3$

$= \frac{1}{3}(3)^3 - 9(3) = 9 - 27 = -18$

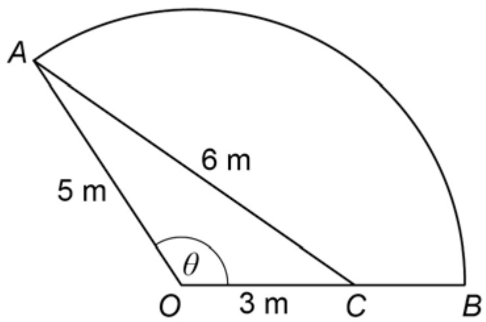
Area is positive, so the area is 18.

Find the area of the shaded region.
Circle your answer.

[1 mark]

–18   –6   6   18

2 A wooden frame is to be made to support some garden decking. The frame is to be in the shape of a sector of a circle. The sector *OAB* is shown in the diagram, with a wooden plank *AC* added to the frame for strength. *OA* makes an angle of θ with *OB*.



2 (a) Show that the exact value of $\sin\theta$ is $\frac{4\sqrt{14}}{15}$

[3 marks]

Use the cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$6^2 = 3^2 + 5^2 - 2(3)(5)\cos\theta$

$$\cos\theta = \frac{3^2 + 5^2 - 6^2}{2(3)(5)} = -\frac{1}{15}$$

But we want the value for $\sin\theta$, so we use the following identity: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \left(\frac{-1}{15}\right)^2$$

$$\sin^2\theta = \frac{224}{225}$$

$$\sin\theta = \frac{4\sqrt{14}}{15}$$

2 (b) Write down the value of θ in radians to 3 significant figures.

[1 mark]

$$\theta = \sin^{-1}\left(\frac{4\sqrt{14}}{15}\right) =$$

2 (c) Find the area of the garden that will be covered by the decking.

[2 marks]

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 1.64$$

$$= 20.5 \text{ m}^2$$

3 A circular ornamental garden pond, of radius 2 metres, has weed starting to grow and cover its surface.

As the weed grows, it covers an area of A square metres. A simple model assumes that the weed grows so that the rate of increase of its area is proportional to A.

3 (a) Show that the area covered by the weed can be modelled by

$$A = Be^{kt}$$

where B and k are constants and t is time in days since the weed was first noticed.

[4 marks]

The rate that the weed grows is $\frac{dA}{dt}$.

We are told this is proportional to A, so

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

Now solve, $\int \frac{1}{A}\, dA = \int k\, dt$

$$\ln A = kt + c$$

$$A = e^{kt+c}$$

$$A = e^{c}e^{kt}$$

$$A = Be^{kt}$$

3 (b) When it was first noticed, the weed covered an area of 0.25 m^2. Twenty days later the weed covered an area of 0.5 m^2

3 (b) (i) State the value of B.

[1 mark]

At $t=0$, $A = 0.25 = Be^0 = B$

so $B = 0.25$

3 (b) (ii) Show that the model for the area covered by the weed can be written as

$$A = 2^{\frac{t}{20} - 2}$$

[4 marks]

When $t=20$, $A=0.5$

$\Rightarrow 0.25e^{20k} = 0.5$

$e^{20k} = 2$

$20k = \ln 2$

$k = \frac{\ln 2}{20}$

So, $A = 0.25e^{\frac{\ln 2}{20}t}$

$A = 0.25\left(e^{\ln 2}\right)^{\frac{t}{20}}$

$A = 0.25(2)^{\frac{t}{20}}$

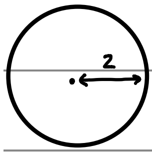
$A = 2^{-2} \times 2^{\frac{t}{20}}$

$A = 2^{\frac{t}{20} - 2}$

Question 3 continues on the next page

3 (b) (iii) How many days does it take for the weed to cover half of the surface of the pond?

[2 marks]



Area $= \pi \times 2^2 = 4\pi$

$\frac{4\pi}{2} = 2^{\frac{t}{20}-2} \Rightarrow 2\pi = 2^{\frac{t}{20}-2}$

$\frac{t}{20} - 2 = \log_2(2\pi)$

$\frac{t}{20} = 2 + \log_2(2\pi)$

$t = 20(2 + \log_2(2\pi))$

$t = 93.03$ days

3 (c) State one limitation of the model.

[1 mark]

The area covered by weed cannot continue to increase as the model suggests It can only grow as big as the pond.

3 (d) Suggest one refinement that could be made to improve the model.

[1 mark]

Growth increases / decreases as it gets bigger.

4 $\int_1^2 x^3 \ln(2x)\,dx$ can be written in the form $p\ln 2 + q$, where p and q are rational numbers.

Find p and q.

[5 marks]

Use integration by parts:

$$\int_1^2 x^3 \ln(2x)\,dx$$

$u = \ln(2x)$ $\quad \frac{dv}{dx} = x^3$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = \frac{1}{4}x^4$

$$= \left[\frac{1}{4}x^4 \ln(2x)\right]_1^2 - \int_1^2 \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{4}(2)^4 \ln(4) - \frac{1}{4}\ln(2) - \frac{1}{4}\int_1^2 x^3\,dx$$

$$= 4\ln(2^2) - \frac{1}{4}\ln 2 - \frac{1}{4}\left[\frac{x^4}{4}\right]_1^2$$

$$= 8\ln 2 - \frac{1}{4}\ln 2 - \frac{1}{4}\left[\frac{16}{4} - \frac{1}{4}\right]$$

$$= \frac{31}{4}\ln 2 - \frac{15}{16}$$

$p = \frac{31}{4}$, $q = \frac{-15}{16}$

5 (a) Find the first three terms, in ascending powers of x, in the binomial expansion of $(1+6x)^{\frac{1}{3}}$

[2 marks]

$$(1+6x)^{\frac{1}{3}} = 1 + \frac{1}{3}(6x) + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)(6x)^2}{2}$$

$$= 1 + 2x - \frac{2 \times 36x^2}{9 \times 2}$$

$$= 1 + 2x - 4x^2$$

5 (b) Use the result from part **(a)** to obtain an approximation to $\sqrt[3]{1.18}$ giving your answer to 4 decimal places.

[2 marks]

$(1+6x)^{\frac{1}{3}} = \sqrt[3]{1.18} \Rightarrow 1+6x = 1.18 \Rightarrow 6x = 0.18 \Rightarrow x = 0.03$

Substitute $x=0.03$ into the answer from part a):

$$\sqrt[3]{1.18} \approx 1 + 2(0.03) - 4(0.03)^2$$

$$\approx 1.0564$$

5 (c) Explain why substituting $x = \frac{1}{2}$ into your answer to part **(a)** does not lead to a valid approximation for $\sqrt[3]{4}$.

[1 mark]

We need $|6x| < 1$, so $|x| < \frac{1}{6}$.

$x = \frac{1}{2}$ does not satisfy this so is not valid.

6 Find the value of $\int_1^2 \frac{6x+1}{6x^2-7x+2}\, dx$, expressing your answer in the form $m\ln 2 + n\ln 3$, where m and n are integers.

[8 marks]

$$\frac{6x+1}{6x^2-7x+2} = \frac{A}{(3x-2)} + \frac{B}{(2x-1)}$$

$$6x+1 = A(2x-1) + B(3x-2)$$

Let $x = \frac{1}{2}$: $\quad 4 = -\frac{1}{2}B$

$B = -8$

Let $x = \frac{2}{3}$: $\quad 5 = \frac{1}{3}A$

$A = 15$

$$\int_1^2 \frac{6x+1}{6x^2-7x+2}\, dx = \int_1^2 \frac{15}{3x-2} + \int_1^2 \frac{-8}{2x-1}\, dx$$

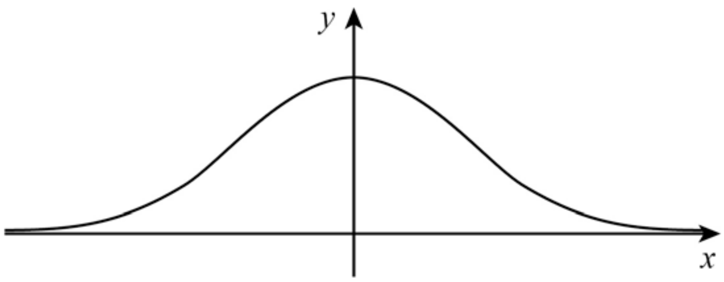
$$= \Big[5\ln(3x-2)\Big]_1^2 - \Big[4\ln(2x-1)\Big]_1^2$$

$$= (5\ln 4 - 5\ln 1) - (4\ln 3 - 4\ln 1)$$

$$= 5\ln(2^2) - 4\ln 3$$

$$= 10\ln 2 - 4\ln 3$$

7 The diagram shows part of the graph of $y = e^{-x^2}$



The graph is formed from two convex sections, where the gradient is increasing, and one concave section, where the gradient is decreasing.

7 (a) Find the values of x for which the graph is concave.

[4 marks]

The gradient is decreasing when $\frac{d^2y}{dx^2} < 0$.

$y = e^{-x^2}$

$\frac{dy}{dx} = -2xe^{-x^2}$ $\qquad \frac{d^2y}{dx^2} = -2x(-2xe^{-x^2}) - 2e^{-x^2}$

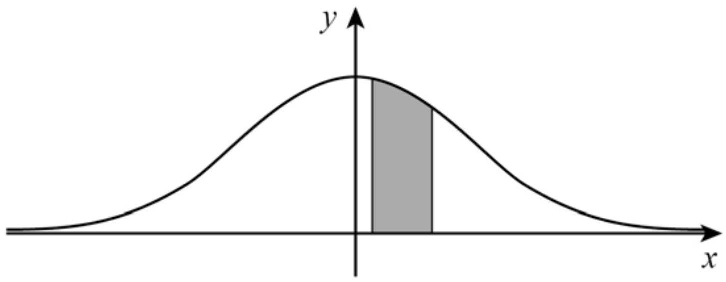
$4x^2e^{-x^2} - 2e^{-x^2} < 0$

$4x^2 - 2 < 0$

$x^2 < \frac{1}{2}$

$\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

7 (b) The finite region bounded by the x-axis and the lines x = 0.1 and x = 0.5 is shaded.



Use the trapezium rule, with 4 strips, to find an estimate for $\int_{0.1}^{0.5} e^{-x^2}\,dx$

Give your estimate to four decimal places.

[3 marks]

$\int_{0.1}^{0.5} e^{-x^2}\,dx \approx \frac{0.1}{2}\left(e^{-0.01} + e^{-0.25} + 2\left(e^{-0.04} + e^{-0.09} + e^{-0.16}\right)\right)$

$= 0.3611$

Question 7 continues on the next page

7 (c) Explain with reference to your answer in part **(a)**, why the answer you found in part **(b)** is an underestimate.

[2 marks]

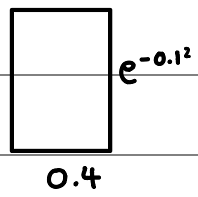
The area we are trying to find falls in the range $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$,

So the trapezia lie below the curve and you get an underestimate.

7 (d) By considering the area of a rectangle, and using your answer to part **(b)**, prove that the shaded area is 0.4 correct to 1 decimal place.

[3 marks]

Take the rectangle



The area of this is $0.4 \times e^{-0.1^2} = 0.396...$

The true area lies somewhere between this value and our value from part b.

So $0.3611 < x < 0.396$.

This means $x = 0.4$ to 1.d.p.

END OF SECTION A
TURN OVER FOR SECTION B

Section B

Answer **all** questions in the spaces provided.

8 Edna wishes to investigate the energy intake from eating out at restaurants for the households in her village.

She wants a sample of 100 households.

She has a list of all 2065 households in the village.

Ralph suggests this selection method.

> "Number the households 0000 to 2064. Obtain 100 different four-digit random numbers between 0000 and 2064 and select the corresponding households for inclusion in the investigation."

8 (a) What is the population for this investigation?

Circle your answer.

[1 mark]

Edna and Ralph	The 2065 households in the village	The energy intake for the village from eating out	The 100 households selected

8 (b) What is the sampling method suggested by Ralph?

Circle your answer.

[1 mark]

Opportunity	Random number	Continuous random variable	Simple random

9 A survey has found that, of the 2400 households in Growmore, 1680 eat home-grown fruit and vegetables.

9 (a) Using the binomial distribution, find the probability that, out of a random sample of 25 households in Growmore, exactly 22 eat home-grown fruit and vegetables.

[2 marks]

$p = \frac{1680}{2400} = 0.7$

$X \sim B(25, 0.7)$

$P(X=22) = \binom{25}{22} \times 0.7^{22} \times 0.3^{3}$

$= 0.02428...$

$= 0.0243$

9 (b) Give a reason why you would **not** expect your calculation in part **(a)** to be valid for the 25 households in Gifford Terrace, a residential road in Growmore.

[1 mark]

The gardens here are likely to be similar, so the gardens are not independent.

10 Some information from the Large Data Set is given in Figures 1 and 2 below.

Figure 1

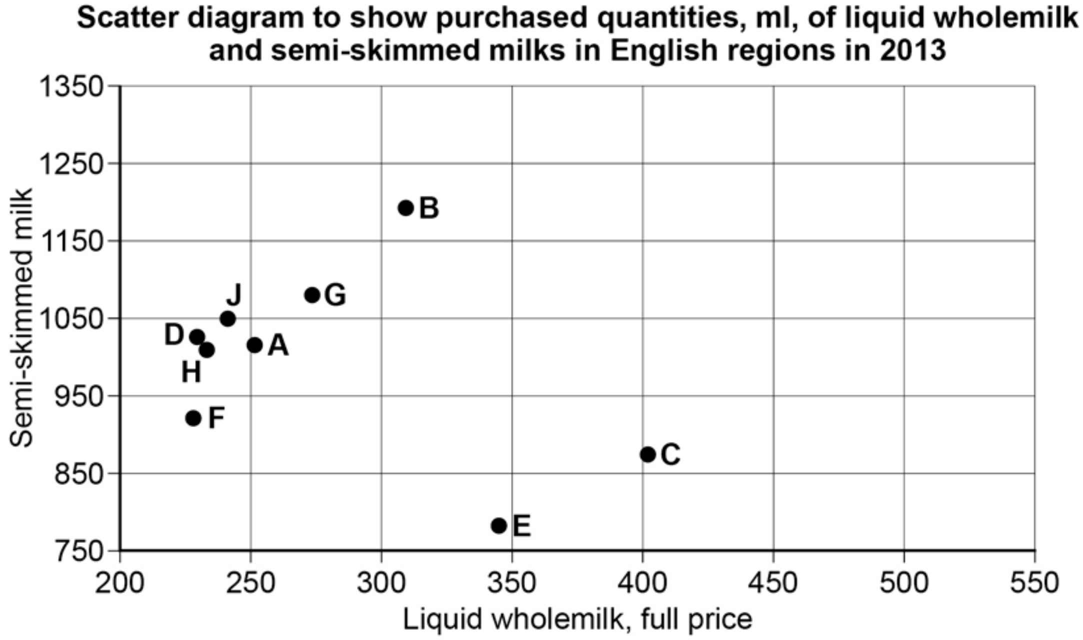
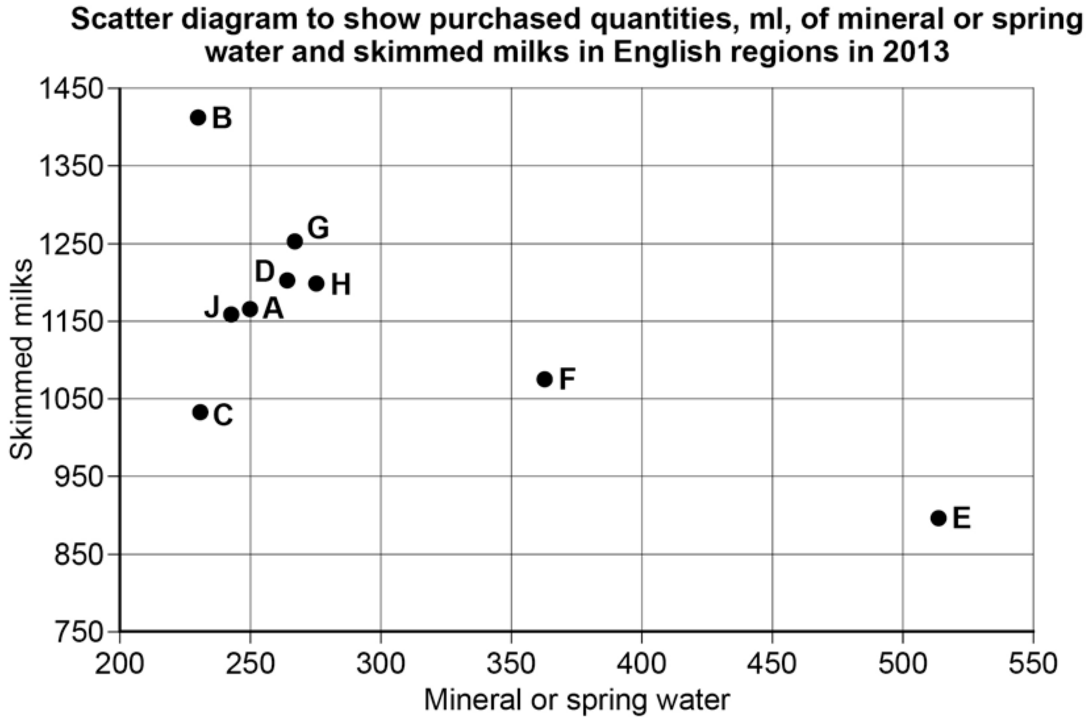


Figure 2



10 (a) Give a reason why the recorded vertical data values are higher for each region in Figure 2 than in Figure 1

[1 mark]

Semi-skimmed milk is a category of skimmed milks, so the value for just semi-skimmed will always be lower than for skimmed.

10 (b) (i) Describe the correlation between 'Semi-skimmed milk' and 'Liquid wholemilk, full price'.

[2 marks]

Regions C and E do not follow the same pattern as the others. If you exclude them then you have a positive correlation between the rest of the data points. So there is a strong positive correlation between purchases of 'semi-skimmed milk' and 'liquid wholemilk, full price'.

10 (b) (ii) Bilal claims that Figure 2 indicates that when people drink more mineral or spring water they tend to drink less skimmed milk.

Comment on Bilal's claim.

[2 marks]

There is a negative correlation between purchases of 'mineral or spring water' and 'skimmed milks'.

However, the study was done with different regions, it tells us nothing about the purchases of individuals.

Question 10 continues on the next page

10 (c) Suggest, with a reason, which region is indicated by the letter E. Use your knowledge of the Large Data Set to support your answer.

[2 marks]

London.

A clear trend in the Large Data set is that London is often an outlier in most categories.

11 Terence owns a local shop. His shop has three checkouts, at least one of which is always staffed.

A regular customer observed that the probability distribution for N, the number of checkouts that are staffed at any given time during the spring, is

$$P(N = n) = \begin{cases} \frac{3}{4}\left(\frac{1}{4}\right)^{n-1} & \text{for } n = 1, 2 \\ k & \text{for } n = 3 \end{cases}$$

11 (a) Find the value of k.

[1 mark]

$\frac{3}{4}\left(\frac{1}{4}\right)^0 + \frac{3}{4}\left(\frac{1}{4}\right)^1 + k = 1$

$\frac{3}{4} + \frac{3}{16} + k = 1$

$k = \frac{1}{16}$

11 (b) Find the probability that a customer, visiting Terence's shop during the spring, will find at least 2 checkouts staffed.

[2 marks]

$P(N \geq 2) = P(N=2) + P(N=3)$

$= \frac{3}{16} + \frac{1}{16}$

$= \frac{1}{4}$

12 During the 2006 Christmas holiday, John, a maths teacher, realised that he had fallen ill during 65% of the Christmas holidays since he had started teaching.

In January 2007, he increased his weekly exercise to try to improve his health.

For the next 7 years, he only fell ill during 2 Christmas holidays.

12 (a) Using a binomial distribution, investigate, at the 5% level of significance, whether there is evidence that John's rate of illness during the Christmas holidays had decreased since increasing his weekly exercise.

[6 marks]

$$\left.\begin{array}{l} H_0 : p = 0.65 \\ H_1 : p < 0.65 \end{array}\right\} X \sim B(7, p)$$

X = number of Christmas holidays John fell ill since January 2007

Assume $X \sim B(7, 0.65)$

$P(X \leq 2) = 0.0556$

$0.0556 > 0.05$ so accept H_0, there is insufficient evidence to suggest he falls ill less.

12 (b) State **two** assumptions, regarding illness during the Christmas holidays, that are necessary for the distribution you have used in part **(a)** to be valid.

For **each** assumption, comment, in context, on whether it is likely to be correct.

[4 marks]

1. Illness each year is indepent of the other years

This assumption is valid as christmases are far apart so illness is unlikely to span that long.

2. The probability of falling ill remains constant.

This is not very valid as your age affects how susceptible you are to illness.

Turn over for the next question

13 In the South West region of England, 100 households were randomly selected and, for each household, the weekly expenditure, £X, per person on food and drink was recorded.

The maximum amount recorded was £40.48 and the minimum amount recorded was £22.00

The results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 3046.14 \qquad \sum (x-\bar{x})^2 = 1746.29$$

13 (a) (i) Find the mean of X

Find the standard deviation of X

[2 marks]

$\bar{x} = \frac{3046.14}{100} = 30.4614$

$s = \sqrt{\frac{1746.29}{99}} = 4.2$

13 (a) (ii) Using your results from part **(a)(i)** and other information given, explain why the normal distribution can be used to model X.

[2 marks]

$\bar{x} \pm 3s = (30.4614 \pm 3(4.2))$

$= (18, 43)$

All data points lie in this range so normal distribution is acceptable.

13 (a) (iii) Find the probability that a household in the South West spends less than £25.00 on food and drink per person per week.

[1 mark]

$P(X<25) = P\left(\frac{X-30.4614}{4.2} < \frac{25-30.4614}{4.2}\right)$

$= P(Z < -1.3)$

$= 0.0967$

13 (b) For households in the North West of England, the weekly expenditure, £Y, per person on food and drink can be modelled by a normal distribution with mean £29.55

It is known that $P(Y < 30) = 0.55$

Find the standard deviation of Y, giving your answer to one decimal place.

[3 marks]

$P\left(Z < \frac{30-29.55}{\sigma}\right) = 0.55$

$\frac{30-29.55}{\sigma} = 0.1257$

$\sigma = \frac{30-29.55}{0.1257}$

$\sigma = 3.57995...$

$\sigma = 3.6$

Turn over for the next question

14 A survey during 2013 investigated mean expenditure on bread and on alcohol.

The 2013 survey obtained information from 12 144 adults.

The survey revealed that the mean expenditure per adult per week on bread was 127p.

14 (a) For 2012, it is known that the expenditure per adult per week on bread had mean 123p, and a standard deviation of 70p.

14 (a) (i) Carry out a hypothesis test, at the 5% significance level, to investigate whether the mean expenditure per adult per week on bread changed from 2012 to 2013.

Assume that the survey data is a random sample taken from a normal distribution.

[5 marks]

$H_0: \mu = 123$

$H_1: \mu \neq 123$

Under H_0: $X \sim N\left(123, \frac{70^2}{1244}\right)$

$P(X > 127) = P\left(\frac{X-123}{\frac{70}{\sqrt{12144}}} > \frac{127-123}{\frac{70}{\sqrt{12144}}}\right)$

$= P(Z > 6.297) = 3 \times 10^{-10}$

$3 \times 10^{-10} < 0.025$

So reject H_0, there is significant evidence to suggest that the mean expenditure has changed.

14 (a) (ii) Calculate the greatest and least values for the sample mean expenditure on bread per adult per week for 2013 that would have resulted in acceptance of the null hypothesis for the test you carried out in part **(a)(i)**.

Give your answers to two decimal places.

[2 marks]

The critical value is 1.96

$123 \pm 1.96 \times \frac{70}{\sqrt{12144}} = [121.75, 124.25]$

So least value is 121.75, largest value is 124.25

14 (b) The 2013 survey revealed that the mean expenditure per adult, per week on alcohol was 324p.

The mean expenditure per adult per week on alcohol for 2009 was 307p.

A test was carried out on the following hypotheses relating to mean expenditure per adult per week on alcohol in 2013.

$H_0 : \mu = 307$

$H_1 : \mu \neq 307$

This test resulted in the null hypothesis, H_0, being rejected.

State, with a reason, whether the test result supports the following statements:

14 (b) (i) the mean UK expenditure on alcohol per adult per week increased by 17p from 2009 to 2013;

[2 marks]

This statement is not supported. We only know that the mean changed but we don't know by how much

14 (b) (ii) the mean UK consumption of alcohol per adult per week changed from 2009 to 2013.

[2 marks]

This is not supported. Expenditure has increased but this doesn't necessarily mean that consumption has too.

15 A sample of 200 households was obtained from a small town.

Each household was asked to complete a questionnaire about their purchases of takeaway food.

A is the event that a household regularly purchases Indian takeaway food.

B is the event that a household regularly purchases Chinese takeaway food.

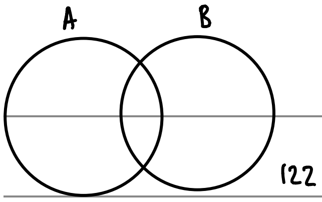
It was observed that $P(B|A) = 0.25$ and $P(A|B) = 0.1$

Of these households, 122 indicated that they did **not** regularly purchase Indian or Chinese takeaway food.

A household is selected at random from those in the sample.

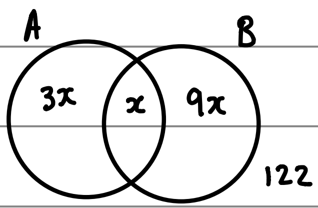
Find the probability that the household regularly purchases **both** Indian and Chinese takeaway food.

[6 marks]



In the A circle, the ratio of households in just A to households in $A \cap B$ is 3:1, because 25% of the households that eat Indian also eat Chinese.

In the B circle, we get a ratio of 9:1 for just B to $A \cap B$. Putting this together we get



$3x + x + 9x = 200 - 122$

$13x = 78$

$x = 6$ So, $P(A \cap B) = \frac{6}{200} = 0.03$

END OF QUESTIONS